International Journal of Thermophysics, Vol. 12, No. 1, 1991

Surface Tension and Viscosity from Damped Free Oscillations of Viscous Droplets

P. V. R. Suryanarayana[1] **and Y. Bayazitoglu**[1]

Received February 12, 1990

Damped oscillations of a viscous droplet in vacuum or in an inert gas of negligible density are considered. The dependence of the complex decay factor on the properties of the liquid is investigated for the first time, and numerical results are compared with earlier studies for special cases. A new method is developed to determine both surface tension and viscosity from a single experiment in which the damping rate and frequency of oscillations are measured. The procedure to determine surface tension and viscosity from oscillating levitated liquids is outlined, and results are presented for various modes of shape oscillations.

KEY WORDS: levitation; oscillating droplets; surface tension; viscosity.

1. INTRODUCTION

Levitation methods, in particular electromagnetic and/or acoustic levitation, have often been suggested as a viable option for containerless processing of materials in reduced gravity environments [1]. In addition to possessing many technological advantages, levitation enables closer and better controlled examination of interfacial dynamics and thermal processes in liquid droplets. Levitation methods have become very attractive in the case of high-temperature materials and melts, due to the added complication of crucible contamination, which they effectively eliminate. Electromagnetic and acoustic levitation are the default choices, owing to their simplicity, stability, and high-temperature capabilities. Thus, a whole new class of thermophysical-property measurements and studies that utilize levitation

[1] Mechanical Engineering and Materials Science Department, Rice University, Houston, Texas 77251, U.S.A.

0195-928X/91/0100-0137$06.50/0

has evolved in recent times [2–7], and many properties that have not been measurable in the past have been measured.

In the past, analysis of damped oscillations of levitated liquids has been suggested as a means to obtain viscosity and surface tension data for liquids. Oscillations of electromagnetically levitated liquids have already been used to measure surface tension [6], and shape oscillations of acoustically levitated liquids have been suggested as a means to measure both surface tension and viscosity [7]. Despite such references to the possibility of measuring viscosity and surface tension from damped oscillations, there appears to be no work in the literature which develops such methods and presents such a technique. The hiatus is partly because the analysis of damped oscillations involves Bessel functions of complex arguments, and numerical solution is usually difficult and has not been reported in the literature so far.

In this paper, we develop a method to measure simultaneously the viscosity and the surface tension from a knowledge of the frequency of oscillations (ω) and the rate of oscillation damping (τ) of a liquid droplet. First, the dynamics of a freely oscillating viscous droplet are studied, and the dependence of the complex decay factor on properties is investigated. Then, a procedure is described that takes ω and τ as inputs and predicts the surface tension γ and the kinematic viscosity ν for any given liquid droplet of known density and geometry, oscillating in a known mode.

2. THEORY OF DAMPED OSCILLATIONS OF A LIQUID DROPLET

The study of the oscillations of liquid droplets is not new and has been undertaken, in one form or the other, from the times of Kelvin [8]. An understanding of the dynamics of oscillating droplets has many applications, notably in chemical engineering [9–11], multiphase flows [12], and for the development of new techniques for thermophysical property measurement [4–7]. A general analysis of small oscillations of a viscous fluid immersed in another liquid has been presented by Miller and Scriven [9]. The special case of a viscous droplet oscillating in a vacuum or a low-density gas has been considered by Reid [13] and summarized by Chandrashekar in his treatise [14]. However, in all these works, solutions for damping rate have been discussed in detail only in the limiting cases of high viscosity or low viscosity. In order to develop a general measurement method, it is necessary to consider the finite viscosity case, and obtain solutions for the damping rate and oscillation frequency. This is attempted in this section.

Consider a liquid droplet of density ρ and equilibrium radius R, executing small shape oscillations about its spherical shape. For simplicity, we assume that there are no external forces acting on the droplet and that the interface is free of contamination and surface reactants. For very low-amplitude oscillations, in the absence of gravity, the perturbation equation governing the flow is

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{-1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} \tag{1}$$

where $\mathbf{v}$ is the velocity vector, t the time, p the pressure, and ν the kinematic viscosity of the liquid. The convection terms can be assumed negligible because they are second order in the velocity. Applying the normal mode analysis to this equation [9, 14], a solution to Eq. (1) can be easily obtained as

$$\tilde{r} w_{nm} = e^{-\beta t} W_{nm}(\tilde{r})\, Y_{nm}(\theta, \phi) \tag{2a}$$

and

$$\tilde{r} \tilde{z}_{nm} = e^{-\beta t} Z_{nm}(\tilde{r})\, Y_{nm}(\theta, \phi) \tag{2b}$$

with $\tilde{r}$ as the radial coordinate. Here β is the complex decay factor, Y_{nm} are the spherical harmonics, and w and $\tilde{z}$ are the radial components of the velocity and vorticity, respectively. The functions of $\tilde{r}$ are found to be

$$W_n(\tilde{r}) = a_1 \tilde{r}^n + a_2 \tilde{r}^{-n-1} + a_3 (\pi/2q)^{1/2} L^{(1)}_{n+1/2}(q) + a_4 (\pi/2q)^{1/2} L^{(2)}_{n+1/2}(q) \tag{3a}$$

and

$$Z_n(\tilde{r}) = b_1 (\pi/2q)^{1/2} L^{(1)}_{n+1/2}(q) + b_2 (\pi/2q)^{1/2} L^{(2)}_{n+1/2}(q) \tag{3b}$$

where $q = \sqrt{(\beta/\nu)}\, \tilde{r}$, n is the mode, and the L's are an appropriate pair of half-integral-order Bessel functions. Applying the kinematic boundary condition, the tangential stress and the normal force balance conditions and simplifying (a detailed discussion of the solution method is given in Refs. 9 and 14 and is not repeated here), we obtain the characteristic equation for the droplet in question as

$$\frac{\omega_n^{*2}}{\beta_n^2} = \frac{2(n^2-1)}{z^2 - 2z Q^J_{n+1/2}} - 1 + \frac{2n(n-1)}{z^2} H \tag{4}$$

where $z=q$ at $\tilde{r}=R$, $Q^J_{n+1/2}(z)$ is the ratio of spherical Bessel functions of successive order,

$$Q^J_{n+1/2}(z)=\frac{J_{n+3/2}(z)}{J_{n+1/2}(z)} \tag{5a}$$

$$H=1-\frac{(n+1)\,Q^J_{n+1/2}(z)}{z/2-Q^J_{n+1/2}(z)} \tag{5b}$$

and ω_n^* is the natural frequency of oscillations of an inviscid droplet oscillating in mode n [15],

$$\omega_n^{*2}=\frac{n(n-1)(n+2)\gamma}{\rho R^3} \tag{6}$$

In what follows, we drop the subscript n for convenience, using it only when there is a possibility of ambiguity.

Equation (4) is the characteristic equation that is to be solved for a complex $\beta=\tau\pm i\omega$. As mentioned earlier, it has been solved only at the limits of high viscosity ($z\to 0$) and low viscosity $z\to\infty$ [9, 14]. Chandrasekhar [14] also includes a detailed discussion of the aperiodic damping factors for a viscous droplet. In both these works, the need for a solution in the complex domain is emphasized but not undertaken due to the numerical difficulties involved in solving the awkwardly transcendental function of a complex variable. Here we give a solution and outline a procedure which is used in the measurement of surface tension and viscosity, described in Section 3.

2.1. Solution for Complex Decay Factor

Using the polar form for the complex variable z

$$z=x+iy=r(\cos\theta+i\sin\theta)$$

in Eq. (4), simplifying, and separating into real and imaginary parts, we obtain

$$\begin{aligned}\alpha^4[r\cos\theta-2Q_{\mathrm{RE}}(z)]=&-r^5\cos 5\theta+2Q_{\mathrm{RE}}(z)\,r^4\cos 4\theta\\&-2Q_{\mathrm{IM}}(z)\,r^4\sin 4\theta+2(2n^2-n-1)\,r^3\cos 3\theta\\&-4n(n-1)(n+2)(r^2\cos 2\theta Q_{\mathrm{RE}}-r^2\sin 2\theta Q_{\mathrm{IM}})\end{aligned} \tag{7a}$$

and

$$\alpha^4[r \sin\theta - 2Q_{IM}(z)] = -r^5 \sin 5\theta + 2Q_{RE}(z)\, r^4 \sin 4\theta + 2Q_{IM}(z)\, r^4 \cos 4\theta + 2(2n^2 - n - 1)\, r^3 \sin 3\theta - 4n(n-1)(n+2)(r^2 \sin 2\theta Q_{RE} + r^2 \cos 2\theta Q_{IM}) \tag{7b}$$

In Eqs. (7a) and (7b), α is defined as

$$\alpha^2 = \omega^* \frac{R^2}{\nu} \tag{7c}$$

and Q_{RE} and Q_{IM} are the real and imaginary parts of the Bessel function ratio defined by Eq. (5) for the complex variable z.

The Bessel function ratio is computed using a continued fraction algorithm with error improvement for the determination of spherical Bessel functions of complex arguments [16]. The algorithm uses a novel technique of evaluating continued fractions that eliminates large storage requirements. The algorithm was checked using the trigonometric expansions for spherical Bessel functions of complex argument [17], for $n = 2$ and 3, and found to be very accurate. The problem is then posed as a minimization problem of function of two variables, and a modified quasi-linearization algorithm is used to find iteratively the values of r and θ that satisfy Eqs. (7) simultaneously. Once r and θ are known, z and hence the complex decay factor β can be obtained easily.

3. RESULTS AND DISCUSSION

3.1. Dynamics of Damped Oscillations

Results for τ and ω are obtained for $n = 2$, 3, 4, and 5 and are presented in Table I and Fig. 1. In Fig. 1, the solid lines represent the nondimensional damping rate (τ/ω^*), and the dashed lines represent the nondimensional oscillation frequency (ω/ω^*) in that mode. As can be seen, oscillations begin only beyond an α^2_{crit}, which increases with increasing n. This α^2_{crit} corresponds to the α^2_{max} that Chandrasekhar [14] derives for aperiodic damping. For $\alpha^2 < \alpha^2_{crit}$, there are two real roots of Eq. (7a), and Eq. (7b) goes identically to zero, i.e., two aperiodic modes of decay exist, one a rapidly decaying mode, and the other a slowly decaying one. Once $\alpha^2 > \alpha^2_{crit}$ or, in other words, the viscosity is less than the critical viscosity corresponding to α^2_{crit}, the modes of decay are characterized by complex β's. As $\nu \to 0$ ($\alpha^2 \to \infty$), the nondimensional oscillation frequency approaches 1,

Table I. The Damped Free Oscillations in a Viscous Droplet

α^2	$\frac{\tau}{\omega_n^*}$	$\frac{\omega}{\omega_n^*}$	α^2	$\frac{\tau}{\omega_n^*}$	$\frac{\omega}{\omega_n^*}$
	$n=2$			$n=3$	
3.69020[a]	0.96286[a]	0.00000	8.82117[a]	0.92544[a]	0.00000
3.80000	0.93980	0.22987	9.00000	0.90640	0.18065
4.00000	0.89330	0.37175	9.20000	0.88740	0.25869
4.20000	0.85125	0.46009	9.40000	0.86923	0.31474
4.60000	0.77818	0.57534	10.00000	0.81915	0.42930
5.00000	0.71687	0.65036	10.60000	0.77484	0.50561
5.40000	0.66471	0.70381	11.80000	0.70000	0.60626
6.00000	0.59961	0.76036	12.60000	0.65821	0.65224
7.00000	0.51614	0.81990	13.80000	0.60482	0.70328
8.00000	0.45375	0.85661	15.40000	0.54693	0.75087
9.00000	0.40541	0.88117	18.00000	0.47543	0.80088
10.00000	0.36687	0.89859	22.60000	0.39029	0.85088
11.00000	0.33544	0.91151	33.00000	0.28622	0.90331
12.00000	0.30932	0.92144	37.00000	0.26140	0.91515
13.40000	0.27939	0.93202	40.00000	0.24580	0.92253
14.80000	0.25518	0.94002	46.00000	0.22006	0.93451
17.40000	0.22059	0.95075	50.00000	0.20591	0.94096
21.20000	0.18516	0.96111	57.00000	0.18526	0.95010
27.00000	0.14976	0.97106	68.00000	0.16019	0.96058
36.00000	0.11624	0.98008	84.00000	0.13395	0.97058
42.00000	0.10131	0.98386	100.00000	0.11514	0.97703
59.00000	0.07440	0.99005	120.00000	0.09800	0.98235
65.00000	0.06805	0.99136	140.00000	0.08533	0.98590
75.00000	0.05958	0.99300	180.00000	0.06785	0.99027
79.80000	0.05623	0.99362	190.00000	0.06455	0.99102
	$n=4$			$n=5$	
15.44109[a]	0.88933[a]	0.00000	23.56980[a]	0.86322[a]	0.00000
16.00000	0.86057	0.22740	24.00000	0.84956	0.15737
16.80000	0.82225	0.34219	24.50000	0.83369	0.22796
17.60000	0.78752	0.41705	25.50000	0.80388	0.31900
18.40000	0.75590	0.47296	27.50000	0.75105	0.43127
19.20000	0.72701	0.51724	30.50000	0.68536	0.53265
20.00000	0.70052	0.55354	33.50000	0.63204	0.59803
20.80000	0.67615	0.58398	36.00000	0.59477	0.63767
21.60000	0.65365	0.60997	39.00000	0.55675	0.67421
24.00000	0.59554	0.66947	45.00000	0.49682	0.72574
26.40000	0.54845	0.71125	53.00000	0.43914	0.77057
31.20000	0.47697	0.76648	66.00000	0.37593	0.81724

[a] Aperiodic damping occurs for coefficients of kinematic viscosity greater than that which corresponds to this entry (i.e., this entry corresponds to $\alpha^2_{\rm crit}$ and $\tau_{\rm crit}$).

Table I. *(Continued)*

α^2	$\frac{\tau}{\omega_n^*}$	$\frac{\omega}{\omega_n^*}$	α^2	$\frac{\tau}{\omega_n^*}$	$\frac{\omega}{\omega_n^*}$
36.80000	0.41808	0.80673	74.00000	0.34770	0.83794
49.60000	0.33399	0.86001	91.00000	0.30216	0.87111
67.20000	0.26779	0.90073	115.00000	0.25641	0.90279
73.60000	0.25040	0.91108	125.00000	0.24129	0.91257
81.60000	0.23175	0.92184	135.00000	0.22785	0.92089
90.40000	0.21428	0.93151	147.50000	0.21301	0.92965
100.00000	0.19802	0.94007	167.50000	0.19290	0.94077
115.00000	0.17705	0.95041	187.50000	0.17625	0.94930
134.50000	0.15565	0.96010	215.00000	0.15754	0.95815
161.50000	0.13336	0.96920	240.00000	0.14369	0.96420
193.00000	0.11430	0.97616	272.50000	0.12897	0.97015
199.00000	0.11128	0.97720	297.50000	0.11956	0.97371

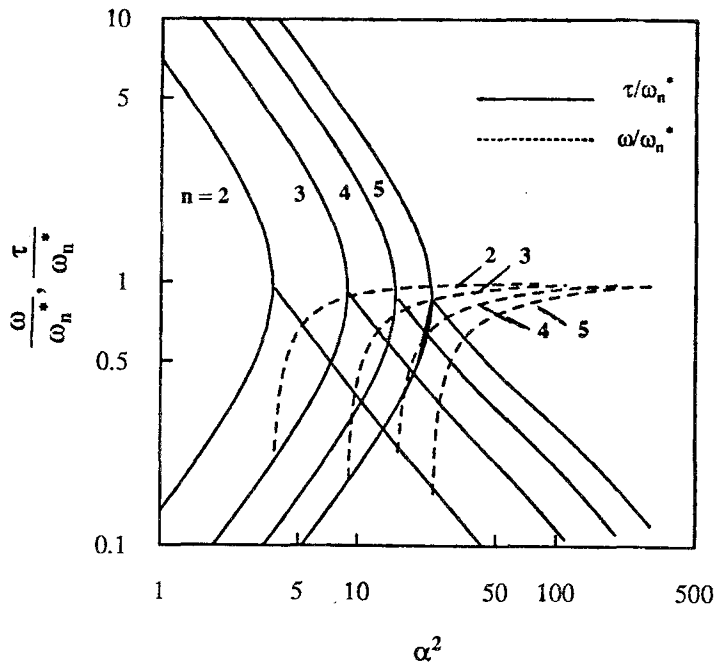

Fig. 1. Damped free oscillations in a viscous droplet: The dependence of damping rate and frequency of oscillations on properties. The ordinate measures the nondimensional damping rate and frequency of oscillations, with respect to the inviscid frequency in that mode. The abscissa measures α^2.

i.e., the frequency ω approaches the inviscid frequency, ω^*. At the same time, the damping rate reduces and asymptotically approaches $(n-1)(2n+1)\,\omega^*/\alpha^2$. These limiting results have already been obtained [14], and our numerical results agree with them, as can be deduced from the limiting values of Table I.

From Fig. 1, it is apparent that for each value of τ below a critical damping rate τ_{crit}, there are two values of α^2, one below and the other above α^2_{crit}. As we move from the aperiodic damping regime to the damped oscillations regime, at first there are two modes of aperiodic decay, and past α^2_{crit}, damped oscillations occur with slow decay, characterized by $\tau < \tau_{crit}$. This value of τ_{crit} is the highest rate of damping possible for damped oscillations and, like α_{crit}, is constant for given mode.

3.2. Determination of Surface Tension and Viscosity

Figure 1 and Table I, in the form presented, afford valuable insight into the dynamics of damped free oscillations of a droplet. However, they cannot directly be used to determine the viscosity and surface tension of the droplet from τ and ω. In this section, we attempt to develop a means of achieving this.

In a typical experiment, a droplet suspended in a vacuum or a gas of negligible density is set into damped oscillations. In order for this to happen, the viscosity of the liquid must be small enough (or the radius so chosen) that $\alpha^2 > \alpha^2_{crit}$. For example, for water, this reduces to the requirement that $R > 0.23$ mm, which is easily achieved. In actual levitation experiments involving liquid metals, the low viscosity and the typical radius of about 5 mm ensure that damped oscillations occur. High-speed pictures are then taken of the oscillating droplet. These pictures are analyzed using Fourier transform techniques to establish the frequency of oscillations and the damping rate. The mode n is deduced from visual inspection of the shape oscillations of the droplet. In addition, the density ρ and the equilibrium radius R also need to be measured (for example, R can be measured from the pictures, and ρ from the mass of the levitated material).

Thus, at the end of the experiment, τ, ω, and the relevant parameters of the problem are known. It is left to find the surface tension and viscosity from this information and the solution in Section 2.

From the definition of z and β,

$$\beta = \frac{\nu}{R^2}(x+iy)^2 = \tau \pm i\omega$$

where x and y are real. Thus, we have

$$x = [R^2/2v\,(\tau + \sqrt{(\tau^2 + \omega^2)})]^{1/2} \tag{8a}$$

and

$$y = [R^2/2v\,(-\tau + \sqrt{(\tau^2 + \omega^2)})]^{1/2} \tag{8b}$$

where, clearly, only the positive root is acceptable for x and y to be real. Or, in the polar formulation used to obtain Eqs. (7),

$$r = [R^2/v\,\sqrt{(\tau^2 + \omega^2)}]^{1/2} \tag{9a}$$

and

$$\tan\theta = \left(\frac{\sqrt{(\tau^2 + \omega^2)} - \tau}{\sqrt{(\tau^2 + \omega^2)} + \tau}\right)^{1/2} \tag{9b}$$

Thus, once τ and ω are known, θ is completely determined, and $r = r(v)$ in Eqs. (7). Thus, Eqs. (7) can be solved again, this time for an unknown r and α. Once r and α are known, v and γ can be found. A program that takes τ and ω as inputs along with R, n, and ρ or mass and computes the surface tension and viscosity has been developed. If the program is to be used to compute only surface tension values from inviscid oscillations, an artificially low value of τ (e.g., $\tau = 0.01$) can be given to the program. The program is very sensitive to initial guesses of r and α^2, but this is a characteristic of the modified quasilinearization algorithm. A good guess would be the one derived from the known properties at room temperature.

Results are presented in a nondimensional form for $n = 2$, 3, 4, and 5. Table II presents some numerical values of the surface tension to viscosity parameter N_{sv},

$$N_{sv} = \frac{\gamma R}{\rho v^2} = \frac{\alpha^4}{n(n-1)(n+2)} \tag{10a}$$

and the nondimensional viscosity number $\bar{X}$,

$$\bar{X} = \tau \frac{R^2}{v}, \tag{10b}$$

for different values of the nondimensional oscillation parameter (or the nondimensional frequency), $\bar{\omega}$,

$$\bar{\omega} = \frac{\omega}{\sqrt{\omega^2 + \tau^2}} \tag{10c}$$

N_{sv}, $\bar{X}$, and $\bar{\omega}$ are the three nondimensional parameters that define the problem completely.

Table II. Determination of Surface Tension and Viscosity: Dependence of N_{sv} and $\bar{X}$ on $\bar{\omega}$ for $n=2$, 3, 4, and 5

	$n=2$		$n=3$		$n=4$		$n=5$	
$\bar{\omega}$	$10^{-3}N_{sv}$	$\bar{X}$	$10^{-3}N_{sv}$	$\bar{X}$	$10^{-3}N_{sv}$	$\bar{X}$	$10^{-3}N_{sv}$	$\bar{X}$
0.001000	0.001702	3.570216	0.002594	8.151839	0.003311	13.739044	0.003968	20.359205
0.009999	0.001702	3.570216	0.002594	8.151854	0.003312	13.739086	0.003969	20.359293
0.287348	0.001857	3.571770	0.002835	8.164824	0.003626	13.777836	0.004353	20.438944
0.371391	0.001977	3.572979	0.003023	8.174925	0.003873	13.808089	0.004656	20.501310
0.514496	0.002322	3.576426	0.003566	8.203777	0.004588	13.894901	0.005537	20.681112
0.894427	0.008794	3.637097	0.014515	8.718832	0.020160	15.505322	0.026057	24.150473
0.948683	0.018246	3.713712	0.032528	9.341665	0.048190	17.362617	0.064927	27.899883
0.970142	0.032405	3.807543	0.061584	9.981439	0.093680	18.940382	0.127095	30.667978
0.980581	0.051841	3.906799	0.102237	10.502523	0.156326	20.044006	0.212105	32.500710
0.992278	0.144415	4.157348	0.292799	11.420168	0.448006	21.856403	0.607721	35.474335
0.995037	0.234999	4.268935	0.477472	11.756273	0.730915	22.515049	0.991721	36.555492
0.998752	1.033843	4.523284	2.111901	12.511064	3.241015	24.004602	4.403375	39.011181
0.999445	2.417571	4.622794	4.956428	12.813521	7.618382	24.607304	10.359433	40.010056
0.999688	4.393771	4.678798	9.029656	12.985958	13.893579	24.952530	18.902889	40.583599
0.999800	6.966849	4.715748	14.342289	13.100617	22.084124	25.182703	30.058350	40.966530
0.999861	10.139791	4.742421	20.901527	13.183807	32.201797	25.349995	43.842350	41.245094
0.999898	13.914793	4.762822	28.712662	13.247664	44.255230	25.478565	60.267006	41.459320
0.999922	18.293556	4.779069	37.779778	13.298656	58.251099	25.581326	79.341682	41.630615
0.999938	23.277454	4.792400	48.106171	13.340581	74.194786	25.665873	101.073921	41.771599

From Table II, it is apparent that the region $0.9 \leqslant \bar{\omega} \leqslant 1.0$ is of greater interest. In order to give as much detail as possible in this region, the results are plotted with $1-\bar{\omega}$ as the abscissa in Figs. 2–4. Figures 2 and 3 show two regions, $0.9 \leqslant \bar{\omega} \leqslant 1.0$ and $\bar{\omega} < 0.9$. It can be observed from these figures that as $\bar{\omega} \to 0$, N_{sv} reaches a minimum which corresponds to the α^2_{crit} for that n. Below this N_{sv}, damping is aperiodic, as discussed earlier. Also, as

$$\bar{\omega} \to 1, \qquad \omega \to \omega^* \tag{11}$$

Or, in terms of N_{sv}, $\bar{\omega}$, and $\bar{X}$, ω can be expressed as

$$\frac{\omega^2}{\omega^{*2}} = \frac{\bar{\omega}^2}{1-\bar{\omega}^2} \frac{\bar{X}^2}{N_{sv} n(n-1)(n+2)} \tag{12}$$

and Eq. (11) implies that the right-hand side of Eq. (12) approaches one as $\bar{\omega} \to 1.0$. For example, for $\bar{\omega} = 0.999938$, and $n = 2$, from Eq. (12), $(\omega/\omega^*) = 0.99726$, and for $\bar{\omega} = 0.980581$, $(\omega/\omega^*) = 0.959$, i.e., the inviscid approximation gives an error of less than 5% for $\bar{\omega} > 0.98$.

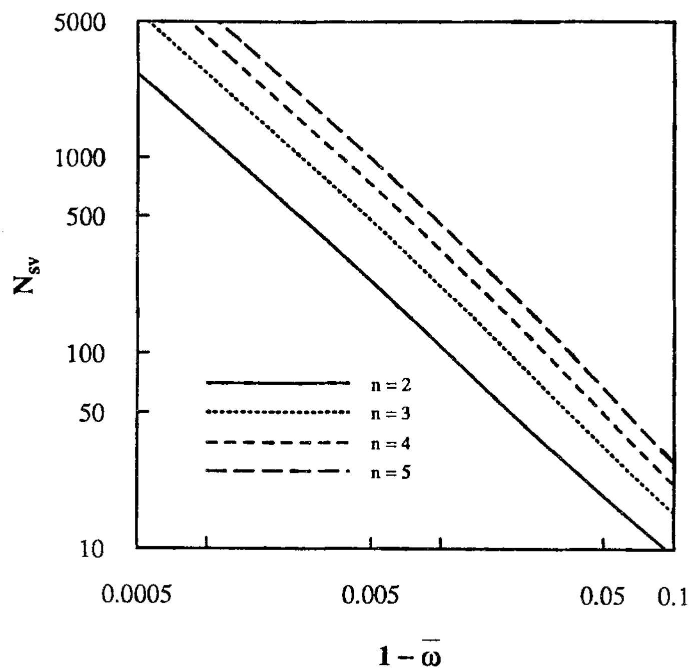

Fig. 2. Dependence of surface tension to viscosity parameter N_{sv} on the damping rate and frequency oscillations: Detail of behavior for $\bar{\omega} > 0.9$. $\bar{\omega}$ is the nondimensional frequency, $\bar{\omega} = \omega^2/\sqrt{\omega^2 + \tau^2}$.

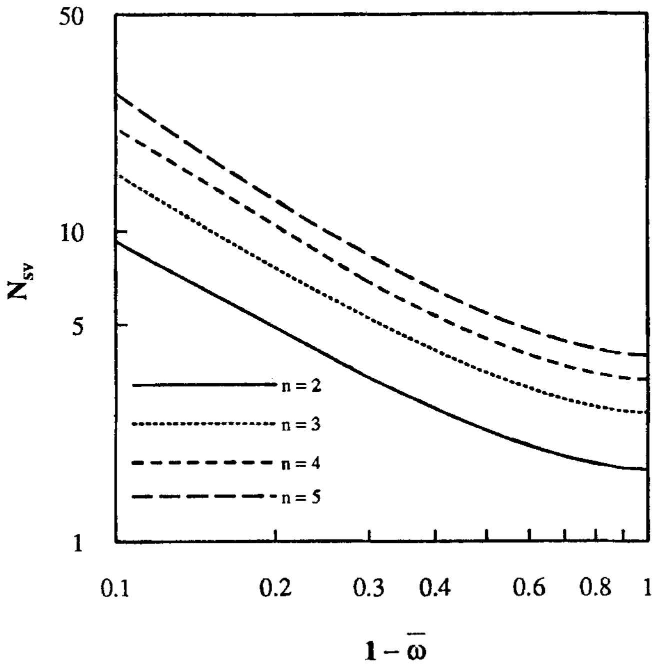

Fig. 3. Dependence of surface tension to viscosity parameter N_{sv} on the damping rate and frequency of oscillations for $\bar{\omega} \leq 0.9$.

Sometimes, damping rate information is difficult to obtain from the experiment (owing to time constraints on the experiment) or not available at all (for example, in the surface tension measurements from oscillation frequency measurements [6]). In such cases, if viscous effects are considerable, an error may result in the property evaluation. However, for mode 2 oscillations, if the viscosity is known, a simple correction can be made to account for the viscous effects when $\bar{\omega} < 0.98$, where inviscid approximation may be inaccurate. This is done as follows.

From Fig. 4, we observe that for $n = 2$, $\bar{X}$ is virtually constant with $\bar{\omega}$ (for $\bar{\omega} < 0.98$, an average $\bar{X}$ may be used: $\bar{X}_{av} = 3.62693$, with a maximum error of 0.27). Thus, if the viscosity of the droplet is known a priori, the constant value of $\bar{X}$ is used to find τ, and this τ may be used in the determination of $\bar{\omega}$. If $\bar{\omega}$ thus calculated is greater than 0.98, the inviscid approximation may be used with less than 5% error. If, however, it is less than 0.98, this value of $\bar{\omega}$ may be used along with Figs. 2 and 3 or Table II to find N_{sv}. from which the surface tension γ can be calculated within the upper and lower bounds of the error in $\bar{X}$. This rather fortunate state of affairs prevails only in the case of mode 2 oscillations, and that, too, if ν

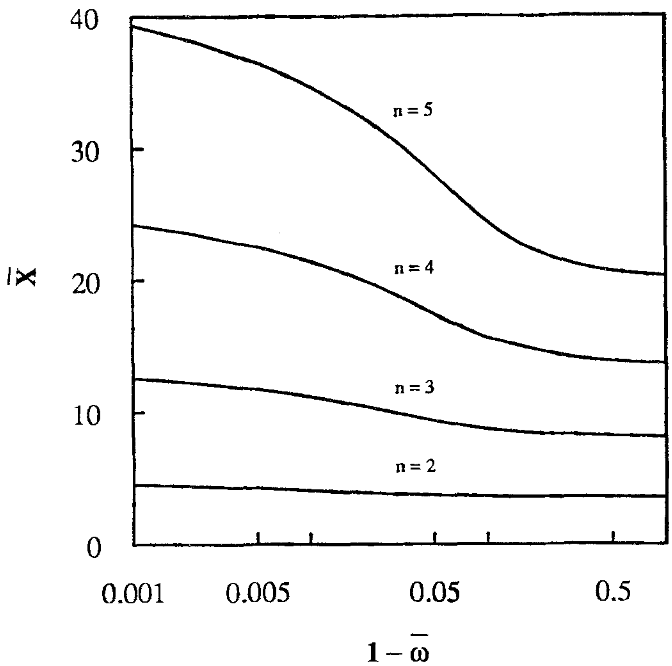

Fig. 4. Dependence of nondimensional viscosity number $\bar{X} = \tau R^2/\nu$ on the damping rate and frequency of oscillations.

is already known. Thus, experimental requirements are somewhat eased, and the results of this work may be applied in cases where viscosity corrections are to be investigated, for example, in the measurement of surface tension using electromagnetic levitation [6].

In summary, the procedure to find surface tension and viscosity from a single experiment may be described as follows:

(1) From experimental observation of an oscillating levitated droplet, the damping rate τ, the frequency of oscillation ω, the radius R, and the density ρ of the droplet are measured. The mode of oscillations n is also noted.

(2) From Table II or Fig. 4, the kinematic viscosity ν is determined from the nondimensional viscosity number $\bar{X}$, $\nu = \tau R^2/\bar{X}$.

(3) Once the viscosity is known, Table II or Figs. 2 and 3 are consulted to obtain the surface tension to viscosity parameter N_{sv}, and from the known viscosity, the surface tension γ is calculated as $\gamma = N_{sv}\nu^2\rho/R$. For mode 2 oscillations, if damping rate is unavailable, a viscosity correction may also be applied as discussed earlier.

4. CONCLUSIONS

A numerical investigation of the damped oscillations of a viscous droplet in vacuum or a gas of negligible density has been performed. A continued fraction approach to the evaluation of the spherical Bessel function ratios, and a modified quasi-linearization approach to the search for zeros of the characteristic equation have been used. The problem is completely defined by three nondimensional parameters, $\bar{\omega}$, N_{sv}, and $\bar{X}$. Using these results, the surface tension and viscosity of liquids can be computed from a knowledge of the damping rate and frequency of oscillations.

It must, however, be remembered that the method outlined in this work applies only to free oscillations. It is also applicable to liquid droplets levitated acoustically or electromagnetically, in reduced-gravity environments where the influence of the external forces and internal flows is minimal. One way to simulate this on earth is by setting a levitated droplet into oscillations and dropping the experiment in a long drop tube. But the effect of levitation forces and internal flows must be accounted for in real applications, before reliable surface tension and viscosity data can be obtained.

ACKNOWLEDGMENT

This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration.

REFERENCES

1. M. Barmatz, *Materials Processing in the Reduced Gravity Environment of Space*, G. E. Rindone, ed. (North-Holland, Amsterdam, 1982), pp. 25–37.
2. C. Y. Ho, ed., *CINDAS Data Series on Material Properties, Vol. I-2*, (Hemisphere, New York, 1986).
3. P. L. Marston and R. E. Apfel, *J. Acoust. Soc. Am.* **67**:27 (1980).
4. Y. Bayazitoglu, P. V. R. Suryanarayana, and U. B. Sathuvalli, *J. Thermophys. Heat Tr.* (in press).
5. G. P. Hansen, S. Krishnan, R. H. Hauge, and J. L. Margrave, *Metallurg. Trans. A* **19A**:1889 (1988).
6. M. E. Fraser, W.-K. Lu, A. E. Hamielec, and R. Murarka, *Met. Trans.* **2**:817 (1971).
7. E. H. Trinh, *Rev. Sci. Instrum.* **56**:2059 (1985).
8. Lord Kelvin, *Mathematical Papers, Vol. 3* (Clay and Sons, London, 1890).
9. C. A. Miller and L. E. Scriven, *J. Fluid Mech.* **32**:417 (1968).
10. L. E. Scriven and C. V. Sternling, *J. Fluid Mech.* **19**:321 (1964).
11. S. S. Bupara, Ph.D. thesis (Department of Chemical Engineering, University of Minnesota, Minneapolis, 1964).

12. V. Levich, *Physiochemical Hydrodynamics* (Prentice Hall, Englewood Cliffs, N.J., 1962).
13. W. H. Reid, *Q. Appl. Math.* **18**:86 (1960).
14. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Clarendon, Oxford, 1961), pp. 467–477.
15. H. Lamb, *Hydrodynamics* (Dover, New York, 1932).
16. W. J. Lentz, *Appl. Optics* **15**:668 (1976).
17. M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).